# Solving Linear Trigonometric Equations in Sine and Cosine

Trigonometric equations are equations that involve trigonometric functions. In many ways, solving a trigonometric equation is very similar to solving polynomial equations or rational equations, where there are only specific values of the variable that are solutions, or there are not solutions at all. However, since trigonometric functions are periodic, solutions are repeated within each period. Thus, sometimes you will be asked to find all solutions (in terms of the period) and other times you will only find solutions in a particular interval. Solving trigonometric equations will require use of the unit circle and the trig values of special angles.

Given a trigonometric equation, solve using algebra.

1) Look for a pattern that suggests an algebraic property, such as the difference of squares or a factoring opportunity.

2) Substitute the trigonometric expression with a single variable, such as or .

3) Solve the equation the same way an algebraic equation would be solved.

4) Substitute the trigonometric expression back in for the variable in the resulting expressions.

5) Solve for the angle.

Examples

1. Find all possible exact solutions for the equation .
2. Solve exactly the following linear equation on the interval :

# Solving Equations Involving a Single Trigonometric Function

When solving trigonometric equations, you will need to make sure you pay attention to which of the six trigonometric functions you are working with. For instance, if you are working with an equation with the tangent function, you will have to use a different period from sine and cosine.

Examples: Solve each of the following equations on the given intervals.

1. ,
2. ,
3. Find all solutions for .
4. ,

# Solve Trigonometric Equations Using a Calculator

Not all values can be found exactly using only the unit circle. When we must solve an equation involving an angle other than one of the special angles, we will need to use a calculator. Make sure it is set in the proper mode (degrees or radians).

Examples: Use a calculator to solve the following equations.

1. , where is in radians

# Solving Trigonometric Equations in Quadratic Form

When a trigonometric equation is written in quadratic form, we can use algebra to solve. If it helps, rewrite the trigonometric equation with a variable such as or . Substitution helps you see the quadratic equation more clearly and then you can solve the quadratic using factoring, quadratic formula, etc.

Examples: Solve each of the following equations on the given intervals.

1. ,
2. ,
3. ,
4. , (Hint: make a substitution to express the equation only in terms of cosine.)

# Solving Trigonometric Equations Using Fundamental Identities

We can also use the fundamental identities to make solving equations simpler.

Examples: Use identities to solve the following equations on the given intervals.

1. ,
2. Find all solutions to the equation:
3. ,

# Solving Trigonometric Equations with Multiple Angles

Sometimes it is not possible to solve a trigonometric equation that have multiple angles, such as or . When confronted with these equations, recall that is a horizontal compression by a factor of 2 of the function . This compression of the graph leads us to believe there may be twice as many intercepts or solutions to compared to .

Examples: Solve each of the following equations on the given intervals.

1. ;
2. ;

# Solving Right Triangle Problems

We can now use all the methods we have learned to solve problems that involve applying the properties of right triangles and the Pythagorean Theorem.

Examples

1. OSHA safety regulations require that the base of a ladder be placed 1 foot from the wall for every 4 feet of ladder length. Find the angle that a ladder of any length forms with the ground and the height at which the ladder touches the wall.
2. A woman is watching a launched rocket currently 11 miles in altitude. If she is standing 4 miles from the launch pad, at what angle is she looking up from horizontal?